AD-A051 053 MASSACHUSETTS INST OF TECH LEXINGTON LINCOLN LAB F/G 9/2

STORE-AND-FORWARD COMPUTER COMMUNICATIONS OVER NOISY CHANNELS. (U)

DEC 77 D J LEINWEBER F19628-78-C-0002

UNCLASSIFIED TR-525 ESD-TR-77-299 NL

| OF |

AD A051053

END

DATE FILMED

4-78

DDC

ESD-TR-77-299

12

Technical Report 525

D. J. Leinweber

Store-and-Forward Computer Communications Over Noisy Channels

2 December 1977

Prepared for the Department of the Air Force
under Electronic Systems Division Contract F19628-78-C-0002 by

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LEXINGTON, MASSACHUSETTS

Approved for public release; distribution unlimited.

The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology, with the support of the Department of the Air Force under Contract F19628-78-C-0002.

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

Raymond L. Loiselle

Raymond L. Loiselle, Lt. Col., USAF
Chief, ESD Lincoln Laboratory Project Office

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

STORE-AND-FORWARD COMPUTER COMMUNICATIONS OVER NOISY CHANNELS

D. J. LEINWEBER

Group 24

TECHNICAL REPORT 525

2 DECEMBER 1977

LEXINGTON MASSACHUSETTS

ABSTRACT*

A model is developed for a computer communications processor with large storage capacity. The model follows single message positive acknowledgment protocol wherein the transmitting node acquires messages from the external environment, transmits them to the receiver, and awaits acknowledgment. Special attention is given to modeling the effects of transmission errors, either in the message or the acknowledgment, upon the service time of the computer communications network. These effects of error have generally been ignored in previous investigations.

In order to realistically account for the service time distributions found in actual computing systems, which have bounded domains, an analysis of M/G/1 queueing servers with service times of this sort was necessary. This analysis (Appendix) focuses upon servers with uniform service times, and those whose service time distributions can be approximated by a section of a normal distribution.

*This document is a condensation of a Ph. D. thesis prepared for the Division of Engineering and Applied Physics at Harvard University. The complete thesis will be available from the Aiken Computation Laboratory, Harvard University, as a technical report entitled "Computer Communications in a Noisy Environment" by early 1978.

CONTENTS

STORE-AND-FORWARD COMPUTER COMMUNICATIONS
OVER NOISY CHANNELS

I. INTRODUCTION

A. Computer Communication Networks

During the 1960's, computer timesharing systems developed from experimental prototypes into sophisticated tools for data processing and for research. Specialized hardware, software, and data resources were available only to users at the sites where they were maintained, while the area of possible utilization of these resources extended far beyond any one installation's immediate proximity. A natural demand for computer communication networks had arisen by the late sixties and early seventies in the computing research community. It was during this same period (in 1969) that the cost of the computers required to dynamically allocate communication facilities dropped below the cost of the facilities being allocated. Thus, for the first time, high-speed networks became cost effective.[1,2]

The ARPANET,[3] a generalized experimental computer network (shown in Fig. I-1), was developed and expanded in the late sixties and seventies. It is now regarded as a mature and successful system and provides daily service to hundreds of users.

Networking has seen application in many areas other than computing research. Government and military users have a continuing need for fast, reliable data communications between a large number of points, as do large business organizations, in retailing, finance, and transportation. Electronic funds transfer systems will increase the national need for fast efficient communications still further; and many systems, both in and out of Government, are currently being deployed, developed, or planned to meet these varied needs. They represent a sizable investment of both money and time, an investment that can be made wisely only when the principles of operation of these complex systems are well understood. It is toward the furtherance of that understanding that this work is directed.

B. Structure of Store-and-Forward Networks

A store-and-forward network consists of a number of physically separated points called nodes, each with a certain storage capacity and communication links to one or (usually) more other points. Messages enter the network nodes from external sources and the communications processor determines which of its neighbors should receive each message, either to deliver it to a destination attached to the node or to forward it to another node which will either deliver it to its final destination or forward it again to other nodes until the destination is reached. The routing decision (which neighbor to forward to) is an important matter, greatly affecting network performance, and has been extensively studied.[4,5]

When a message is transmitted to another node, the sender retains a copy of it in storage until the receiver determines by any of several error detecting schemes[6] that the message is correct and sends back a special acknowledgment message to the transmitting node, thus freeing the storage used to retain a copy of the message, which would be retransmitted if the acknowledgment were not received within a predetermined time-out interval. Errors that occur due to the presence of noise of any real communications channel may be found in the acknowledgments as well as the basic message traffic. These acknowledgment errors will result in the retention in storage and retransmission of messages that have actually been correctly received.

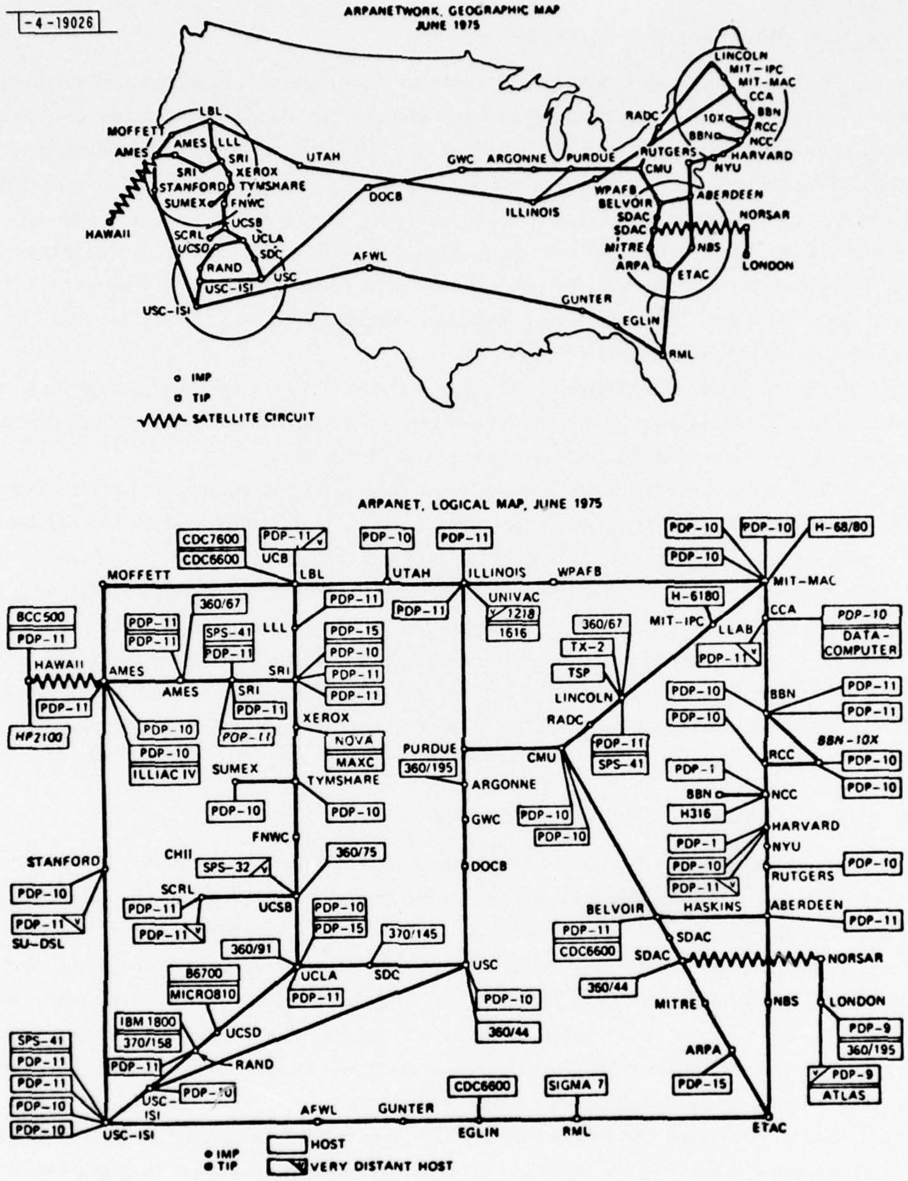

Fig. I-1. ARPANET maps.

C. Nodal Blocking

If messages are arriving at a node at a sufficiently fast rate or errors are requiring that messages be retained for long periods of time, it is possible for all of a node's message storage space to become full. Each message requires a buffer, and if none are available, the node is said to be blocked to incoming message traffic. Buffer space is always reserved, however, for acknowledgments and certain other high-priority types of messages. This is necessary so a blocked node can become unblocked by receiving acknowledgments and resuming normal operations. When the node is blocked, arriving messages are ignored, and therefore not acknowledged. The sender will, after the prescribed time interval, retransmit and wait for an acknowledgment. Thus, nodes transmitting to a blocked node will be retaining more of their messages for retransmission and are thus more likely to become blocked themselves.

D. Previous Work

The manner in which nodes become blocked due to stochastic message arrivals and service completions was analyzed by Schweitzer, Lam, and Closs[7,8,9] by modeling the network as a network of simple queueing servers. They consider transmission errors and nodal blocking to be independent events, a simplifying but incorrect assumption. Their results are useful in the analysis of networks containing very high quality communications channels. They employ an incidence matrix characterization of network routing structure that greatly simplifies calculation of the loading on individual nodes. The lack of consideration of errors in acknowledgments is a serious flaw in this model, though in quiet environments it does provide a useful tool for evaluating the stochastic effects of arrival rates and communications service times.

Propagation of blocking in a network was studied by Ziegler,[10] who modeled each node as a two-state (either blocked or free) Markovian system and studied, both analytically and by means of simulation, the spread of blocking in the net. He found that, as one would suspect, the spread is very rapid and can quickly reduce network throughput to zero. This was without any consideration of the effects of errors.

The severity of the blocking problem has caused designers of actual networks to go to great lengths to circumvent it. Most involve cutting off incoming traffic at the source when potential blocking situations develop, others employ reservation schemes to avoid such situations in advance. A survey of these flow control techniques can be found in Ref. 11. While the methods discussed do avoid blocking, they do so at a high price in additional overhead. It is important to consider the operation of the basic communications processes being used. If sound engineering decisions are made in the initial network design, the various "fixes" that must be employed in a real system will be invoked less frequently. System resources can then be used toward productive ends, rather than to support internal control functions. Most of the analysis of network performance, even very detailed analyses,[12] do not adequately address the problems caused by errors in the transmission and acknowledgment of messages. As will be seen in the following sections, these error rates have a strong effect on the performance of store-and-forward systems. These effects are basic in nature and not the result of any particular algorithm or implementation. The goal here is to further the understanding of the processes underlying the operation of communications networks in order to make possible system designs that increase the amount of useful work the network can perform.

II. THE EXTENDED STORAGE MODEL OF A COMMUNICATIONS NODE

A. General Description and Architectural Considerations

Nearly all communications processors constructed in the past have avoided the use of rotational storage devices for reliability reasons. It was considered highly undesirable to include mechanical components in a device designed to provide uninterrupted service for long periods of time. Thus, the amount of memory available for buffering was often limited by both addressing and economic constraints.

With the increasing availability of cheap LSI memories, large virtual address spaces, and the expected appearance of economical solid-state mass-storage devices using charge-coupled devices,[13] magnetic bubbles,[14,15] or electron beam technologies,[16] it is reasonable to anticipate future communications processor designs with far less stringent restrictions on buffer space. A general discussion of the effect of these new technologies on computer system architectures is found in Ref. 17.

The model discussed here is directly applicable to such large-memory machines, and yields information on buffering requirements that enables one to determine how much space actually constitutes a "large" memory, and what the approximate distribution of buffer utilization will be.

B. Network of Queues Model

Figure II-1 shows the network of queues and servers which constitutes the extended storage model. Messages arrive from external sources with rate λ and queue at the transmission server X, whose service-time distribution has probability density function (pdf) $f_X(x)$. After transmission, messages that arrive without error, and therefore will not be timed out, queue at the acknowledgment server A. A fraction p_1 of the messages transmitted will not be served by the acknowledgment process because of time outs of errors in transmission of the message.

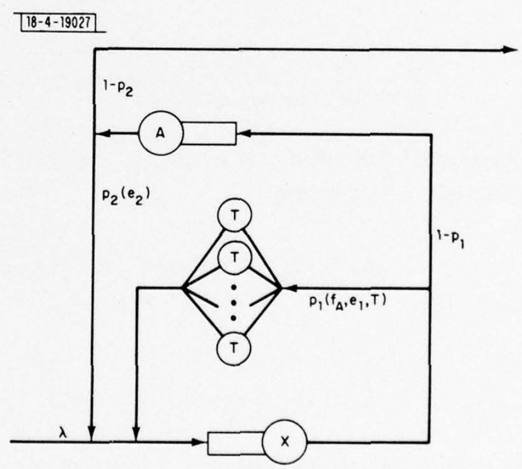

Fig. II-1. The extended storage model.

If $f_A(a)$ is a pdf for the random variable A which denotes acknowledgment service time, and T is the length of the time-out interval, then the fraction of messages that are timed out, f_T, is

$$f_T = \int_T^{\infty} f_A(a)\, da \quad . \tag{II-1}$$

The probability of a bit error on the outgoing link is e_1 and the message length is ℓ_m (bits) so the fraction of messages received with errors, f_m, is

$$f_m = 1 - (1 - e_1)^{\ell_m} \quad . \tag{II-2}$$

The branching probability into the time-out servers (labeled T on Fig. II-1) is then

$$p_1 = f_m + (1 - f_m) f_T \tag{II-3}$$

indicating that all messages with errors and those without errors that are not acknowledged quickly enough will be timed out. The decision tree in Fig. II-2 illustrates this calculation.

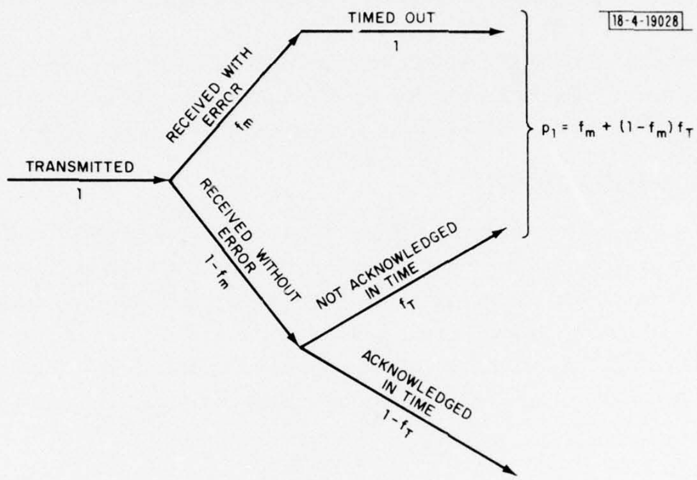

Fig. II-2. Tree for calculating p_1. Events are denoted above branches, probabilities are below.

The functional dependence of p_1 on f_A, e_1, and T is found by substituting the expressions (II-1) and (-2) into (-3) yielding

$$p_1(f_A, e_1, T) = 1 - (1 - e_1)^{\ell_m} \left[1 - \int_T^{\infty} f_A(a)\, da\right] \tag{II-4}$$

or

$$p_1(f_A, e_1, T) = 1 - (1 - e_1)^{\ell_m} \int_0^T f_A(a)\, da \tag{II-5}$$

since f_A is a density defined over a positive domain.

The time-out process is represented by the infinite parallel set of non-queueing servers labeled T, each with a deterministic service time equal to the time-out interval T. This representation is preferable to a queueing server with deterministic service time since real time outs do proceed in parallel. Two messages entering the parallel servers at times 0 and t

(where $t < T$) will be timed out correctly at times T and $t + T$ rather than T and $2T$, as would be the case for an (initially idle) single server. All timed-out messages proceed back into the queue at server X for retransmission.

Messages not destined for time outs follow the upper branch to the acknowledgment server A with probability $1 - p_1$. The distribution of service times at this server is the conditional distribution of $f_A(a)$ given that a is less than the time-out interval T. This is necessary to account for those messages that were correctly received but will be timed out and follow the lower branch to the time-out servers. Denoting this distribution by $g_A(a)$ we have

$$g_A(a) = \frac{f_A(a)}{\int_0^T f_A(a)\,da} \qquad \text{for } a < T$$

$$g_A(a) = 0 \qquad \text{for } a \geqslant T \quad . \tag{II-6}$$

An acknowledgment will be sent by server A over a line with bit error rate e_2. If the acknowledgment is received in error, the message will be retransmitted. This is shown as the lower branch leaving A, with probability p_2 given by

$$p_2 = 1 - (1 - e_2)^{\ell_a} \tag{II-7}$$

where ℓ_a is the length of the acknowledgment in bits. Messages whose acknowledgments are received without error proceed up the second branch with probability $1 - p_2$ and leave the system.

PRECEDING PAGE BLANK - NOT FILMED

III. SOLUTION OF THE EXTENDED STORAGE MODEL

A. Analysis

Networks of the type described in the previous section can be analyzed by using the branching probabilities and external arrival rate to compute the total arrival rate to each element of the system and solving for each element separately. This method is known to be exact for networks containing only exponential servers[18] and has been widely employed in the analysis of complex computer systems.[19,20] The decomposition method has recently been shown to be accurate when applied to closed queueing networks with general service-time distributions,[21] and it is reasonable to expect even greater accuracy when solving open systems (such as the one described in the previous section) since there is less coupling between the various servers.[22]

Let r_x, r_a, and r_t denote the total arrival rates at the transmission, acknowledgment, and time-out servers, respectively. Then, by considering the steady state flows shown in Fig. II-1, we have

$$r_x = \lambda + r_t + P_2 r_a$$

$$r_t = P_1 r_x$$

and

$$r_a = (1 - P_1)\, r_x \quad . \tag{III-1}$$

Solving the above equations for r_x, r_a, and r_t yields

$$r_x = \frac{\lambda}{(P_1 - 1)\, P_2 - P_1 + 1}$$

$$r_t = \frac{\lambda P_1}{(P_1 - 1)\, P_2 - P_1 + 1}$$

and

$$r_a = \frac{\lambda}{1 - P_2} \quad . \tag{III-2}$$

The arrival rates for the two queueing servers x and A can be used along with the distributions $g_A(a)$ and $f_x(x)$ to find the mean and variance of the distribution of the number of customers in the queues or in service at x and A. The formulas derived in the Appendix for a class of finite-domained distributions will be used for this purpose. Let us denote these means and variances by $\overline{N}_a$, $\overline{N}_x$, σ_a^2, and σ_x^2.

The time-out servers must be analyzed separately. Assuming a Poisson arrival process with rate r_t, the number of busy servers will be Poisson distributed with mean Tr_t. Thus, we have

$$\overline{N}_t = Tr_t \tag{III-3}$$

and since the variance of a Poisson random variable is equal to its mean

$$\sigma_t^2 = Tr_t \quad . \tag{III-4}$$

TABLE III-1 TEST PARAMETERS	
Transmission Service Time:	uniform on [0.01, 0.02] sec
Acknowledgment Service Time:	uniform on [0.05, 0.10] sec
Time-out Interval:	0.4 sec
Message Length:	200 bits
Acknowledgment Length:	100 bits

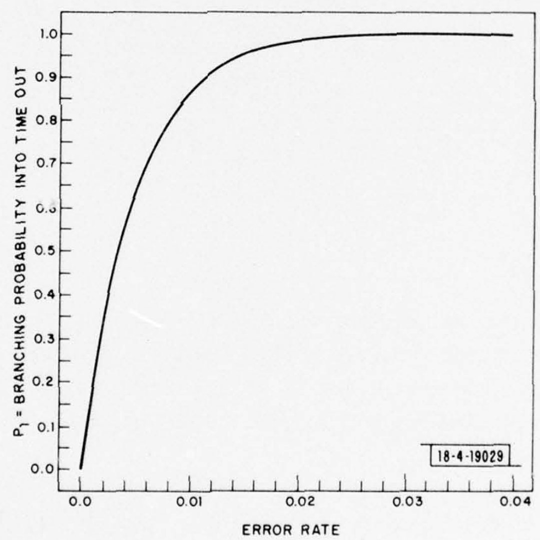

Fig. III-1. Branching probability into time-out servers vs error rate.

The number of buffers in use is

$$N = N_x + N_t + N_a \tag{III-5}$$

with mean

$$\overline{N} = \overline{N}_x + \overline{N}_t + \overline{N}_a \tag{III-6}$$

and variance

$$\sigma_N^2 = \sigma_x^2 + \sigma_t^2 + \sigma_a^2 \quad . \tag{III-7}$$

These two quantities are used along with Chebychev's inequality

$$P_r \, [\,|N - \overline{N}| \geqslant \epsilon] \leqslant \frac{\sigma_N^2}{\epsilon^2} \text{ for } \epsilon > 0 \tag{III-8}$$

to determine how many buffers are required to assure a minimum blocking probability P_{min}. This number, M, is found from (III-8) to be

$$M = \overline{N} + \sqrt{\sigma_N^2 / P_{min}} \quad . \tag{III-9}$$

B. Effects of Error and Arrival Rates on Queue Size and Delay

The results derived in the previous sections will be used to analyze a specific communication process, described in Table III-1.

The error and arrival rates will range from zero to those values beyond which the system will not operate (i.e., contain only finite queues).

An important parameter in the extended storage model is P_1, the branching probability into the time-out servers T. For this example, we have

$$P_1 = 1 - (1 - e_1)^{200}$$

since the upper bound on acknowledgment service times is less than the time-out interval and the integral on the right-hand side of Eq. (II-5) evaluates to one. Figure III-1 shows a plot of P_1 vs error rate. For longer messages the ascent becomes even more rapid. The branching probability P_2 out of the acknowledgment process into retransmission is similar in form. The families of curves shown in Figs. III-2 and -3 illustrate the behavior of a store-and-forward message system in noisy environments ranging from an extremely high quality (a bit error rate of 10^{-5}) to an extreme noise level (10^2) which would not normally be encountered in a commercial application. Note from Fig. III-3 that the expected delay goes up in high-noise situations, even at very low traffic rates. This is due to the near certainty of retransmissions. When these branching probabilities are used with Eqs. (III-1) through (-6) to compute the expected queue size $\overline{N}$ as a function of arrival rate for a range of error rates corresponding to those normally found on noisy commercial facilities (see Fig. III-4), the behavior of $\overline{N}$ can be seen in Fig. III-5. These graphs show the entire arrival range from zero to beyond the saturation point where infinite queueing will occur. Note how this point moves back with increases in error rates. A detailed view of a portion of Fig. III-5 is found in Fig. III-6, showing a more reasonable operating range

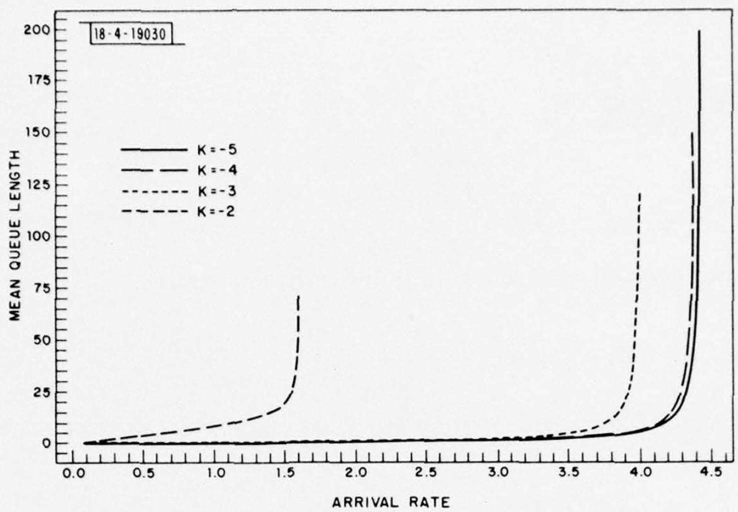

Fig. III-2. Mean queue length vs arrival rate for a wide range of error rates. K is $\log_{10}$ of the error rate.

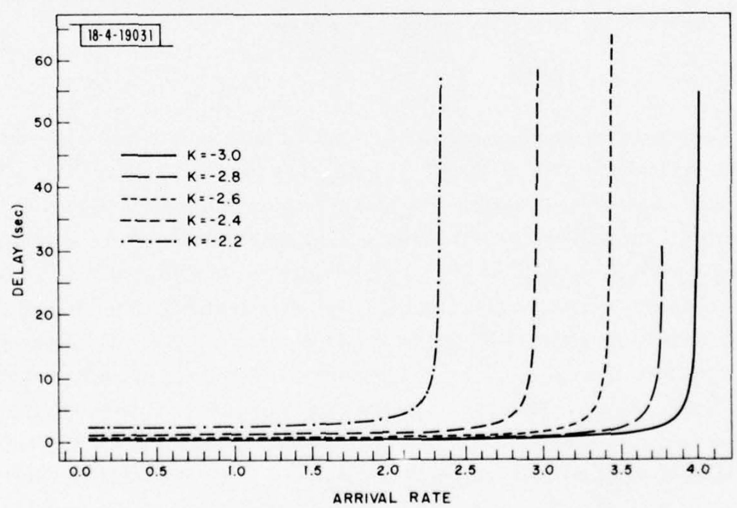

Fig. III-3. Delay vs arrival rate in a very noisy environment. K is $\log_{10}$ of the error rate.

of arrival rates. The effects on message delay can be found by means of Little's formula $\overline{N} = \lambda\overline{W}$, where $\overline{W}$ is the expected delay. The expected waits associated with the queues shown in Figs. III-5 and -6 are shown, respectively, in Figs. III-7 and -8.

An interesting way of looking at the error-rate dependencies shown implicitly in Figs. III-5 through -8 is to hold the arrivals constant and vary instead the error rate. This is done in Figs. III-9 (showing the size) and -10 (showing delay), which clearly show the existence of error level thresholds, corresponding to the saturation points in arrival rates, beyond which the system cannot be operated. These are shown explicitly in Fig. III-11, which plots the maximum allowable error rate against the arrival rate.

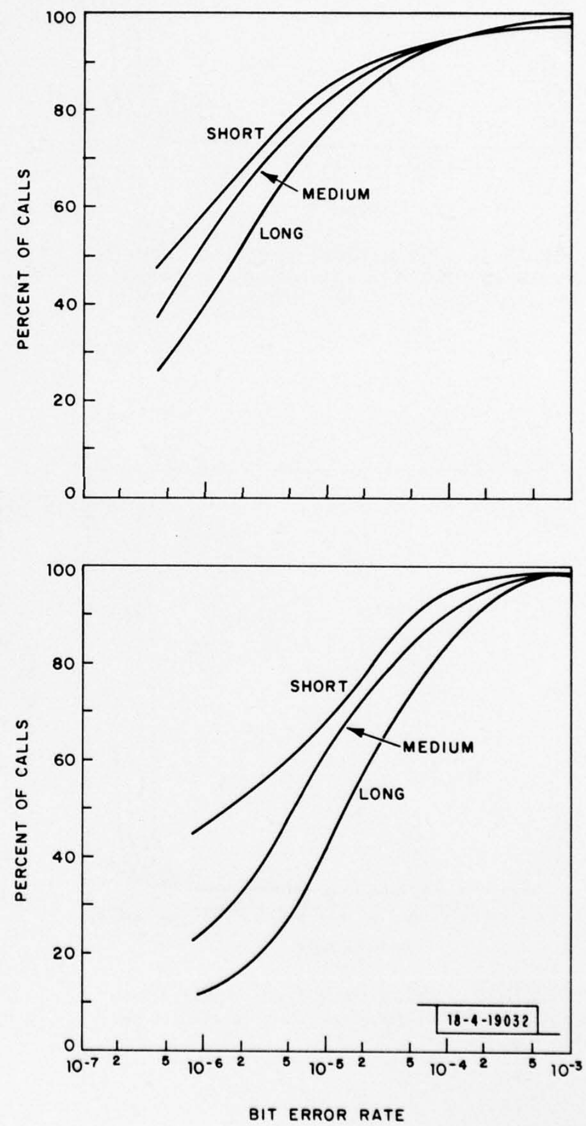

Fig. III-4. Error-rate distributions.

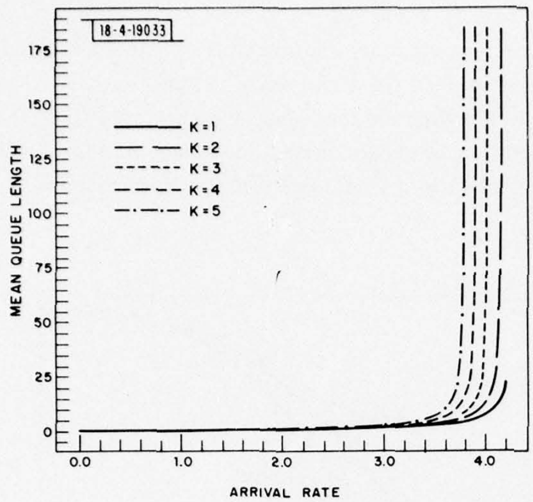

Fig. III-5. Mean queue length vs arrival rates from zero to saturation for a range of error rates $e = ke_0$ with $e_0 = 0.0003$.

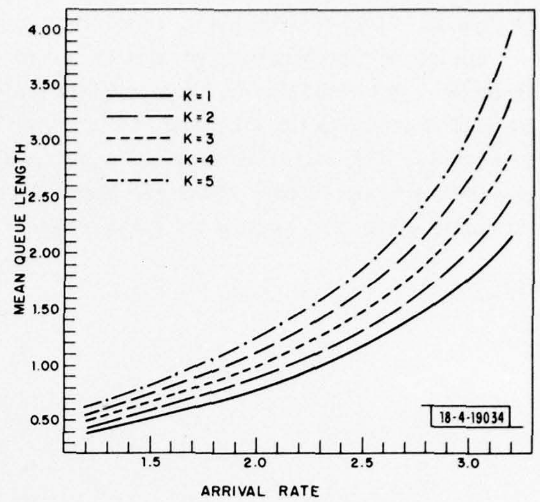

Fig. III-6. Mean queue length vs arrival rates in a normal operating region with error rate e as in Fig. III-5.

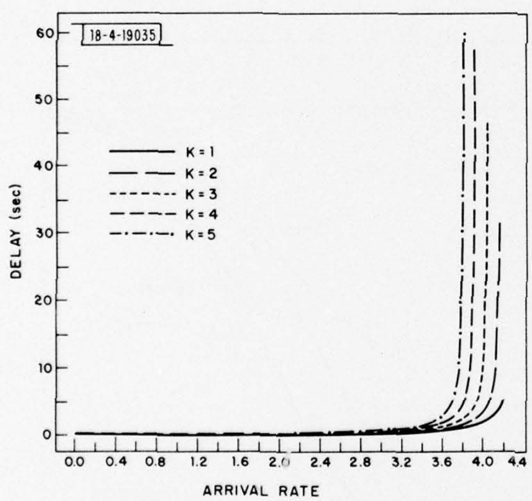

Fig. III-7. Delay vs arrival rates from zero to saturation, with error rate e as in Fig. III-5.

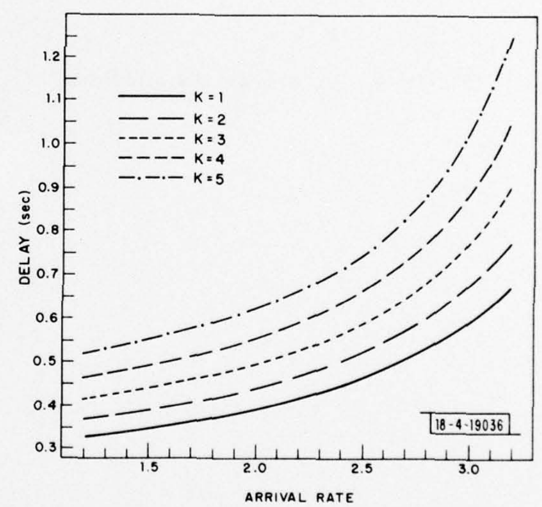

Fig. III-8. Delay vs arrival rates in a normal operating region, with error rate e as in Fig. III-5.

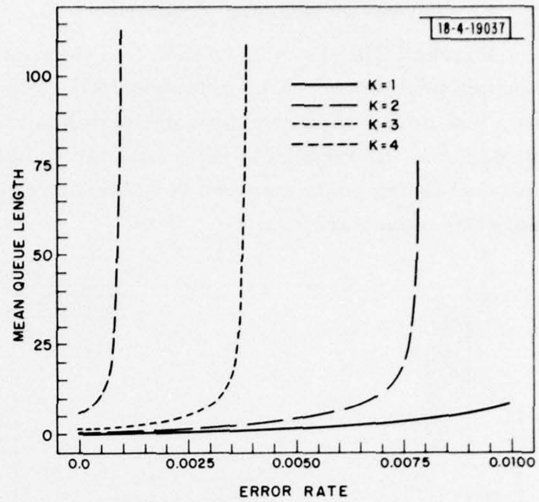

Fig. III-9. Mean queue length vs error rates, zero to saturation, for a range of arrival rates $\lambda = k\lambda_0$ with $\lambda_0 = 1.0$.

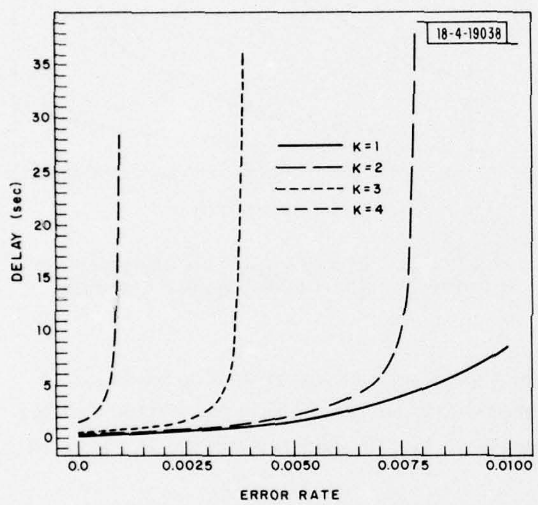

Fig. III-10. Delay vs error rates, zero to saturation, with arrival rates λ as in Fig. III-9.

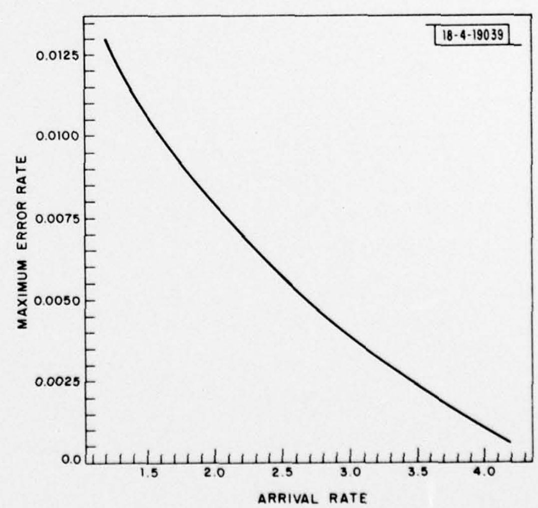

Fig. III-11. Maximum operating error rate vs arrival rate.

C. Bounds on Blocking Probability

Equation (III-9) can be used to find the number of buffers required to assure some specified blocking probability. This relationship is shown, for a range of error rates, in Fig. III-12. Note that to approach zero probability of blocking an asymptotically infinite number of buffers is required. These curves also demonstrate that error rate has a much stronger effect on the required buffer pool size when very low blocking probabilities are involved than when more moderate values are sought.

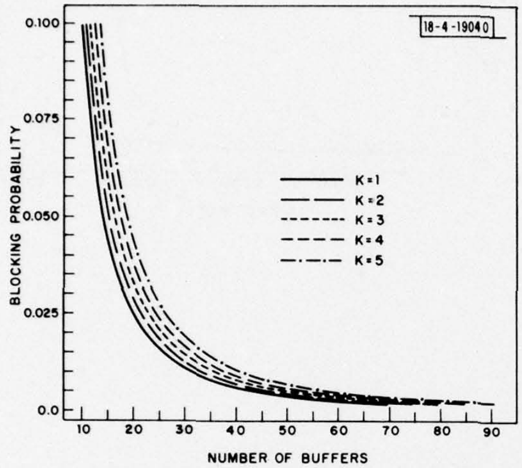

Fig. III-12. Blocking probability vs number of buffers for a range of error rates $e = ke_0$ with $e_0 = 0.0003$.

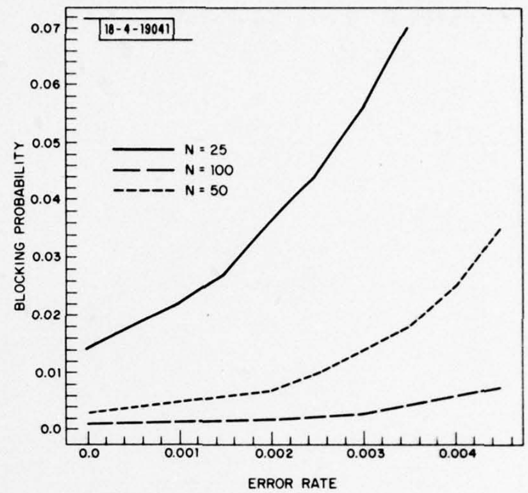

Fig. III-13. Blocking probability vs error rate for 25, 50, and 100 buffer systems.

By looking at the vertical intercept points of the type of curves shown in Fig. III-12, it is possible to find the relationship between blocking probability and error rate for a fixed number of buffers. The result when this is done for systems with 25, 50, and 100 buffers is shown in Fig. III-13.

IV. CONCLUSIONS

A. Error Rates and Computer Communications

Error probabilities are basic properties of all communications systems. They are of fundamental importance in the classical analyses of both analog[23] and digital[24] communications processes.

A model for store-and-forward communications was analyzed with full consideration of error rates, and strong effects on system performance were found to exist. It provides a straightforward way of showing this basic dependence.

The extended storage model makes use of new results derived from the Pollaczek-Khinchin formulas for M/G/1 queues with service-time distributions that are bounded both above and below (see Appendix). This is the case for virtually all real computer systems, but has often been ignored due to the analytical complexity it entails.

The existence of sharp error-rate thresholds, analogous to the arrival-rate bounds found in this research and by others, is a clear demonstration of an important effect that has been previously overlooked. The existence of a region of the error-arrival-rate space beyond which communications nodes cannot operate without incurring infinite queueing delays has been shown, along with a method of finding this region with relative ease.

The option of allocating resources to quieter channels rather than additional memory of processing capacity is often overlooked. Much very recent, and very detailed, work on computer communications systems, such as in Ref. 25, does not consider the effect of communications errors at all. Others, as in Ref. 26, have looked at various design issues with full consideration given to error effects, and found strong dependencies due, in part, to the effects observed here.

Real computer communications systems are, of course, more complex than the models that have been analyzed. Each will differ in its approach (or lack of one) to flow control, routing, reservations, sequencing, and many other functions performed in an actual system. The models developed in this research are not substitutes for simulation or actual testing, but are analytic tools used to show the operating characteristics of more general systems. The models, free of unnecessary software detail, depend only on fundamental physical parameters and yield operating bounds against which actual or proposed designs can be compared. This kind of analysis has an important place in computer communications systems analysis and design.

Further work in this area could include considerations of the bursty nature of errors on actual communications facilities. A more detailed exploration of the multidimensional space of arrivals, errors, time-out intervals, message size distributions, service rates, degree of processor parallelism, and communications protocols could be performed using numerical multivariate optimization techniques, subject to differing cost constraints.

B. Use of Symbolic Manipulation Techniques

MACSYMA, the recently matured symbolic manipulation system at the M.I.T. Artificial Intelligence Laboratory,[27] was used extensively in this work, particularly in the Appendix. When using MACSYMA, one finds the distinction between numerical and analytical solutions becomes somewhat blurred. The symbolic answers to some relatively simple problems can easily fill several pages, and their usefulness in comparison with ordinary numerical methods is questionable.

Intermediate expression swell, that demon which causes symbolic representations to expand manyfold during processing and then shrink back in the final stages of a symbolic operation, is a paradigm for the successful use of the symbolic operations (each with its own case of intermediate expression swell). Once the problem has been formulated, the best use of the symbolic manipulator occurs when the intermediate results of the calculation are large, complex objects which we could not normally deal with, but the final answers take a form simple enough to be useful and comprehensible as algebraic expressions. This was the fortunate case in the analysis of the M/G/1 queueing system in the Appendix. It should be kept in mind when solving problems symbolically that mistakes are as easy to make in MACSYMA as on paper, though they are admittedly of a different character. A useful technique was found to be the incorporation of checking techniques into the calculations that would be far too time consuming to do manually, e.g., making sure that expressions which are supposed to be probability densities integrate to "1.0" or calculating the same quantity by two or more methods and comparing the results.

APPENDIX

M/G/1 QUEUEING SERVERS WITH BOUNDED SERVICE TIME DISTRIBUTIONS

I. INTRODUCTION

Many events in computing systems occur over a bounded time range. Virtually all processing tasks require some minimum amount of time to accomplish and, barring the occasional software idiosyncrasy that can shunt a task off into the low-priority queue for a very long time, each task will be completed (or abnormally terminated) within some reasonable maximum time. In communications, time-sharing, and other real-time systems, the difference between these upper and lower bounds will often be small.

Most of the literature on queueing theory deals with servers having exponential or Erlang distributions, both defined on the positive infinite half interval.

In this appendix, M/G/1 queueing servers having service-time distributions with finite domains will be considered, and the first two moments of the queue size distribution derived.

II. THEORY OF M/G/1 SERVERS

The basis for the analysis of M/G/1 systems was formulated by Pollaczek and Khinchin in the early thirties;[28,29] treatments in contemporary notation can be found in Ref. 2 (Vol. 1) and Ref. 30. The fundamental results will be stated here. Let S, denoting the service time, have a distribution f(s) with finite first, second, the third moments, and let λ denote the intensity of the Poisson arrival process. Then, taking $\rho = \lambda\overline{s}$, the mean number of customers in the system is given by

$$\overline{N} = \rho + \frac{\lambda^2\overline{s^2}}{2(1-\rho)}$$

The z-transform of the distribution of the number of customers is

$$Q(z) = B(\lambda - \lambda z)\,\frac{(1-\rho)\,(1-z)}{B(\lambda-\lambda z) - z}$$

where $B(\lambda - \lambda z)$ refers to the LaPlace transform of the service-time distribution B(w) evaluated at $\lambda - \lambda z$ with

$$B(w) = \int_0^\infty e^{-wx}\, f(x)\, dx \quad .$$

If Q(z) can be readily inverted or written as a power series, the distribution of the number of customers can be found directly, but for most cases of interest this is not possible. The differentiation properties of the z-transform of a distribution are employed to find the moments. The first moment (mean) is $dQ(z)/dz$ evaluated at $z = 1$. (This yields the same result as the earlier mean value formula.) The second moment is derived from the relation

$$\left.\frac{d^2Q(z)}{dz^2}\right|_{z=1} = \overline{N^2} - \overline{N} \quad .$$

Since $z = 1$ is an indeterminate point of $Q(z)$, the evaluation above will require the repeated application of L'Hôpital's rule. This eventually yields the desired expression for the variance of the distribution of the number of customers given below.

$$\sigma_N^2 = \frac{\lambda^2 \overline{s^3}}{3(1-\rho)} + \left(\frac{\lambda^2 \overline{s^2}}{2(1-\rho)}\right)^2 + \frac{\lambda^2 [(3-2\rho)\overline{s^2}]}{2(1-\rho)} + \rho(1-\rho)$$

In all cases, these moments exist only if $\rho < 1$.

Information about the behavior of the distribution is obtained by means of the Chebyshev inequality, which states that, for any $\epsilon > 0$

$$\Pr[|N - \overline{N}| \geqslant \epsilon] \leqslant \frac{\sigma_N^2}{\epsilon^2} \quad .$$

III. UNIFORMLY DISTRIBUTED SERVICE TIMES

The uniform distribution is useful when only upper and lower bounds on the service time are known and no statistical information on its behavior between these bounds is available. Let the lower bound be A and the upper bound be B. Then the distribution takes on the constant value $1/(B-A)$ on the interval $[A, B]$ and has moments given below.

$$\overline{S} = (A + B)/2$$

$$\overline{S^2} = (A^2 + AB + B^2)/3$$

$$\overline{S^3} = (B + A)(B^2 + A^2)/4 \quad .$$

Substitution of these expressions into the formulas of the previous section and some simplification yields, for the mean number in the system,

$$\overline{N} = \frac{(B^2 + 4AB + A^2)\lambda^2 + (-6B - 6A)\lambda}{\lambda(6B + 6A) - 12}$$

for

$$\rho = \lambda(A + B)/2 < 1 \quad .$$

In the case where $A = 0$ and $B = 1$, this simplifies further to

$$\overline{N} = (\lambda^2 - 6\lambda)/(6\lambda - 12) \qquad \text{for } \lambda < 2 \quad .$$

The variance is found to be

$$\begin{aligned}\sigma_N^2 = \lambda^2(&8B^4\lambda^2 + 22AB^3\lambda^2 + 30A^2B^2\lambda^2 + 22A^3B\lambda^2 + 8A^4\lambda^2 \\ &- 3B^4\lambda - 6AB^3\lambda - 12B^3\lambda - 6A^2B^2\lambda - 24AB^2\lambda - 6A^3B\lambda \\ &- 24A^2B\lambda - 54B\lambda - 3A^4\lambda - 12A^3\lambda - 54A\lambda + 6B^3 + 6AB^2 \\ &+ 6A^2B + 6A^3 + 108)/[18(B\lambda + A\lambda - 2)^2] \quad .\end{aligned}$$

This is not as bad as it looks and is easily coded as several Fortran-like statements. For the case where [A, B] = [0, 1] it simplifies to

$$\sigma_N^2 = \frac{8\lambda^4 - 69\lambda^3 + 114\lambda^2}{18\lambda^2 - 72\lambda + 72} \quad .$$

In both instances, the same restrictions on ρ as stated for the mean also hold.

For the special case of constant service times with A = B = C, the resulting expressions for $\overline{N}$ and σ_N^2 are

$$\overline{N} = \frac{C\lambda(C\lambda - 2)}{2(C\lambda - 1)}$$

and

$$\sigma_N^2 = \frac{15C^4\lambda^4 + (-4C^4 - 12C^3 - 18C)\,\lambda^3 + (4C^3 + 18)\,\lambda^2}{12(C^2\lambda^2 - 2C + 1)} \qquad \text{for } \lambda < 1/C \quad .$$

IV. THE BI-TRUNCATED NORMAL DISTRIBUTION

A wide range of useful distributions is obtained by bounding a section of a normal distribution above and below and multiplying by an appropriate normalization constant. There is some intuitive appeal to this distribution. One form of the central limit theorem states that a random variable that is the sum of a large number of independent random variables with finite variances will have a normal distribution. The response of a computer system to a service request certainly depends on a large number of factors and the finite domain properties discussed in the first section assure finite variance. Some of the many forms of distribution in this family are shown in Fig. A-1. All are different sections of a normal distribution with mean equal to 10, only the variance and bounds are changed.

The probability density function for a normal distribution N(m, σ) truncated above at B and below at A is

$$f(s) = G \frac{1}{\sqrt{2\pi}\,\sigma^2} \exp\left[-(s - m)^2/2\sigma^2\right] \qquad \text{for } A \leqslant s \leqslant B \quad , \qquad 0 \text{ elsewhere}$$

where G is found by setting

$$\int_A^B f(s)\, ds = 1$$

to be

$$G = \frac{2}{\operatorname{erf}\left(\frac{m - A}{\sqrt{2}\,\sigma}\right) - \operatorname{erf}\left(\frac{m - B}{\sqrt{2}\,\sigma}\right)}$$

with

$$g(x) = \operatorname{erf}\left(\frac{m - x}{\sqrt{2}\,\sigma}\right)$$

then

$$G = \frac{2}{g(A) - g\,(B)}$$

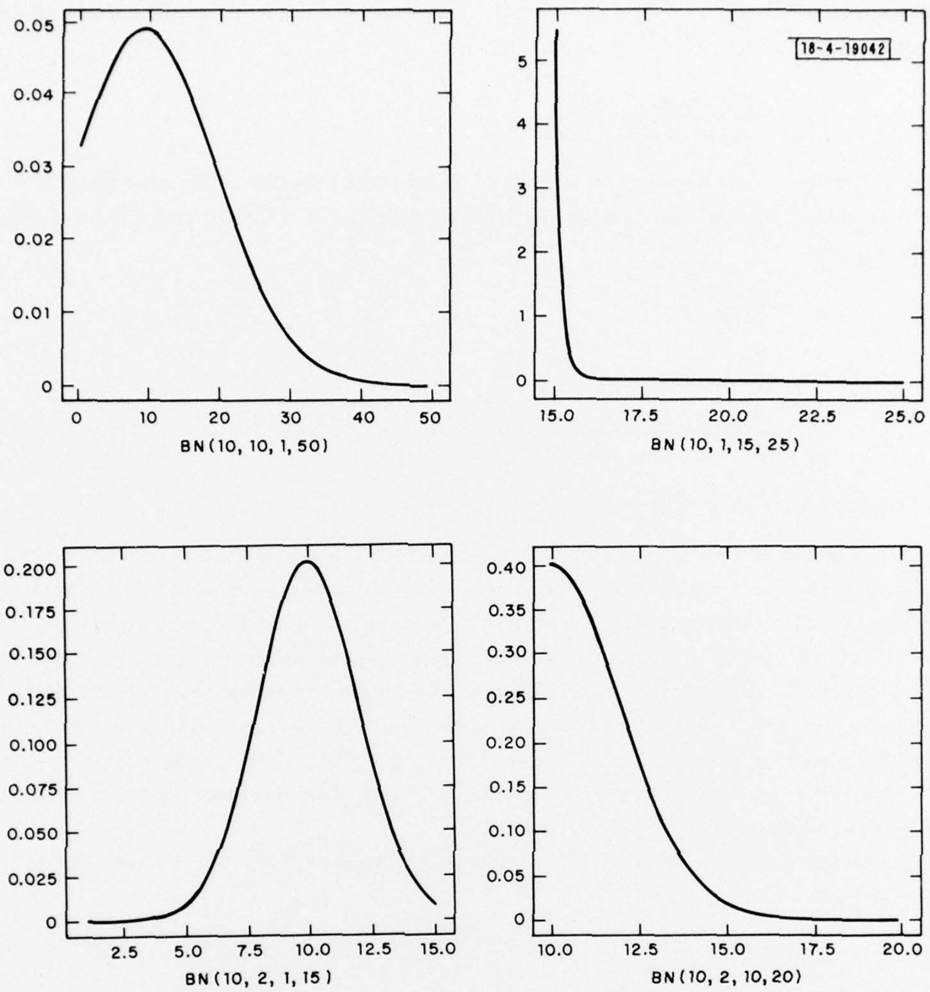

Fig. A-1. Examples of bi-truncated normal distributions, BN(m, s, A, B). (Note changes in scale.)

TABLE A-1			
k	a_k	b_k	c_k
1	mR/C	1	-1
2	$(m^2 + \sigma^2)\ R/C$	$2\sigma\ (m + B)$	$-2\sigma\ (m + A)$
3	$(3m^2 + m^3)\ R/C$	$4\sigma^3 + 2(m^2 + Bm + B^2)\ \sigma$	$-4\sigma^3 + 2(m^2 + Am + A^2)\ \sigma$

The first three moments of the bi-truncated normal distribution BN(m, σ, A, B) where m and σ are the mean and standard deviation of the underlying normal distribution are given below.

$$\overline{S} = \frac{\left(\frac{mR}{C}\,[g(B) - g(A)] + p - q\right)}{\frac{R}{C}\,[g(B) - g(A)]}$$

$$\overline{S^2} = \frac{\frac{(m^2 + \sigma^2)}{C}\,[g(B) - g(A)] + 2\sigma\,[(M + B)\,p + (M + A)\,q]}{\frac{R}{C}\,[g(B) - g(A)]}$$

and

$$\overline{S^3} = \frac{\left[\frac{(3m\sigma^2 + m^3)\,R}{C}\,g(B) - g(A) + [4\sigma^3 + 2(m^2 + Bm + B^2)\,\sigma]\,p - [4\sigma^3 + 2(m^2 + Am + A^2)\,\sigma]\,q\right]}{\frac{R}{C}\,[g(B) - g(A)]}$$

where

$$C = 1/\sqrt{2\pi}$$

$$p = 2\sigma\,\exp[(2Bm + A^2)/2\sigma^2]$$

$$q = 2\sigma\,\exp[(2Am + B^2)/2\sigma^2]$$

and

$$R = \exp[(m^2 + A^2 + B^2/2\sigma^2] \quad .$$

Letting D = g(B) − g(A), these moments can be written as

$$\overline{S^k} = \frac{a_kD + b_kp + c_kq}{RD/C}$$

with a_k, b_k and c_k given in Table A-1.

Evaluation of these nine coefficients and substitution of S^k in the Pollaczek-Khinchin formulas

$$\overline{N} = \rho + \frac{\lambda^2\overline{S^2}}{2(1 - \rho)}$$

and

$$\sigma_N^2 = \frac{\lambda^2\overline{S^3}}{3(1 - \rho)} + \left(\frac{\lambda^2\overline{S^2}}{2(1 - \rho)}\right)^2 \frac{\lambda^2\,[(3 - 2\rho)\,\overline{S^2}]}{2(1 - \rho)} + \rho(1 - \rho)$$

with $\rho = \lambda\overline{s}$ will produce the first two moments of the distribution of the number of customers in an M/BN(m, σ, A, B)/1 queueing system.

Preceding Page Blank - NOT FILMED

ACKNOWLEDGMENTS

The consistent encouragement and support of Dr. Walter I. Wells and other members of the staff of the M.I.T. Lincoln Laboratory were invaluable throughout the course of this research. Useful guidance was provided by Professors Ugo Gagliardi, Harry Lewis, Philip Bernstein, Thomas Cheatham, and Richard Meyer of Harvard University, and Dr. Jeff Buzen of B.G.S., Inc.

REFERENCES

1. L. G. Roberts, "Data by the Packet," IEEE Spectrum 11, 46 (1974).
2. L. Kleinrock, Queueing Systems, Vols. 1 and 2 (Wiley, New York, 1975).
3. R. E. Kahn, "Resource-Sharing Computer Communications Networks," Proc. IEEE 60, 1397 (1972).
4. J. M. McQuillan, "Adaptive Routing Algorithms for Distributed Computer Networks," Ph. D. Thesis, Harvard University, and Bolt, Beranek and Newman Report No. 2381 (May 1974).
5. C. E. Agnew, "On Quadratic Adaptive Routing Algorithms," CACM 19, 18 (1976).
6. H. O. Burton and D. D. Sullivan, "Errors and Error Control," Proc. IEEE 60, 1293 (1972).
7. P. J. Schweitzer and S. S. Lam, "Buffer Overflow in a Store-and-Forward Network Node," IBM Thomas J. Watson Research Center Report RC-5759 (December 1975).
8. S. S. Lam, "Store-and-Forward Buffer Requirements in a Packet Switching Network," IEEE Trans. Commun. COM-24, 394 (1976).
9. F. Closs, "Packet Arrival and Buffer Statistics in a Packet Switching Node," IBM Zurich Research Center Report RZ594 (1973).
10. J. F. Zeigler, "Nodal Blocking in Large Networks," Ph. D. Thesis, UCLA-Eng.-7167 (October 1971).
11. J. Herrmann, "Flow control in the ARPA network," Computer Networks 1, 65 (1976).
12. J. M. McQuillan et al., "Improvements in the design and performance of the ARPA network," Proc. Fall Joint Computer Conf. 41, 741 (1972).
13. H. B. Crouch et al., "CCDs in Memory Systems Move Into Sight," Computer Design 15, No. 9, 75 (1976).
14. A. H. Bobeck, P. I. Bonyhard, and J. E. Geusic, "Magnetic Bubbles – An Emerging New Memory Technology," Proc. IEEE 63, 1176 (1975).
15. J. E. Juliussen, "Magnetic Bubble Systems Approach Practical Use," Computer Design 15, No. 10, 81 (1976).
16. W. C. Hughes et al., "A Semiconductor Nonvolatile Electron Beam Accessed Mass Memory," Proc. IEEE 63, 1230 (1975).
17. J. H. Wensley, "The Impact of Electronic Disks on System Architecture," Computer 8, 44 (1975).
18. J. R. Jackson, "Networks of Waiting Lines," Oper. Res. 5, 518 (1957).
19. J. C. Browne et al., "Hierarchical Techniques for the Development of Realistic Models of Complex Computer Systems," Proc. IEEE 63, 966 (1975).
20. R. M. Brown et al., "Memory Management and Response Time," CACM 20, 153 (1977).
21. A. Shum, "Queueing Models for Computer Systems with General Service Times," Ph. D. Thesis, Harvard University (December 1976).
22. J. Buzen, private communication, April 1977.
23. C. E. Shannon, The Mathematical Theory of Communication (University of Illinois Press, Urbana, 1949).
24. L. E. Franks, Ed., Data Communication: Fundamentals of Baseband Transmission, Benchmark Papers in Electrical Engineering and Computer Science (Dowden, Hutchinson, and Ross, Stroudsburg, Pennsylvania, 1974).
25. J. Spragins, "Simple Derivation of Queueing Formulas for Loop Systems," IEEE Trans. Commun. COM-25, 446 (1977).

26. I. Gitman, "Comparison of Hop-by-Hop and End-to-End Acknowledgment Schemes in Computer Communication Networks," IEEE Trans. Commun. COM-24, 1258 (1976).

27. Mathlab Group, M.I.T. Laboratory for Computer Science, "Macsyma Reference Manual," Version 8 (1975).

28. F. Pollaczek, "Uber eine Aufgabe dev Wahrscheinlichkeisth Theorie," I-II Math. Z. 32, 64 (1930).

29. A. Y. Khinchin, "Mathematical Theory of Stationary Queues," Math. Sbornik 39, 73 (1932).

30. A. O. Allen, "Elements of Queueing Theory for System Design," IBM Syst. J. 2, 161 (1975).

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

18 19

REPORT DOCUMENTATION PAGE — READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: ESD-TR-77-299
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): 6 Store-and-Forward Computer Communications Over Noisy Channels
5. TYPE OF REPORT & PERIOD COVERED: 9 Technical Rep[redacted]t
6. PERFORMING ORG. REPORT NUMBER: Technical Report 525 — 14 TR-525
7. AUTHOR(s): 10 David J. Leinweber
8. CONTRACT OR GRANT NUMBER(s): 15 F19628-78-C-0002
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Lincoln Laboratory, M.I.T., P.O. Box 73, Lexington, MA 02173
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: Program Element 65705F, Project No. 649L (16)
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Systems Command, USAF, Andrews AFB, Washington, DC 20331
12. REPORT DATE: 11 2 Dec[redacted]77
13. NUMBER OF PAGES: ~~32~~ 13 30 p.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): Electronic Systems Division, Hanscom AFB, Bedford, MA 01731
15. SECURITY CLASS. (of this report): Unclassified
15a. DECLASSIFICATION DOWNGRADING SCHEDULE:
16. DISTRIBUTION STATEMENT (of this Report): Approved for public release; distribution unlimited.
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report):
18. SUPPLEMENTARY NOTES: None
19. KEY WORDS (Continue on reverse side if necessary and identify by block number):

computer communications networks	noisy channels	ARPANET
store-and-forward networks	queueing servers	extended storage model

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A model is developed for a computer communications processor with large storage capacity. The model follows single message positive acknowledgment protocol wherein the transmitting node acquires messages from the external environment, transmits them to the receiver, and awaits acknowledgment. Special attention is given to modeling the effects of transmission errors, either in the message or the acknowledgment, upon the service time of the computer communications network. These effects of error have generally been ignored in previous investigations.

In order to realistically account for the service time distributions found in actual computing systems, which have bounded domains, an analysis of M/G/1 queueing servers with service times of this sort was necessary. This analysis (Appendix) focuses upon servers with uniform service times, and those whose service time distributions can be approximated by a section of a normal distribution.

DD FORM 1473, 1 JAN 73 — EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

U.S. GOVERNMENT PRINTING OFFICE: 1978 700-031/139

207650

Alt